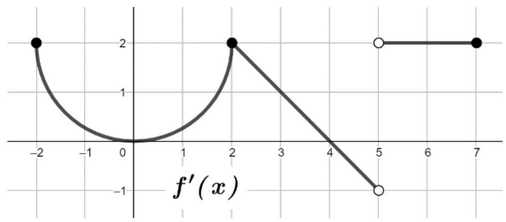
**Activity**: A Journey Through Calculus from A to Z




$$h(x) = \begin{cases} \dfrac{\sin(x-1)}{x-1}, & x < 1 \\ kx^2 - 8x + 6, & 1 \le x \le 3 \\ 4e^{2x-6} - x^2 + 5, & x > 3 \end{cases}$$

| $x$ | $g''(x)$ |
|---|---|
| $-4$ | $13$ |
| $-1$ | $10$ |
| $0$ | $8$ |
| $2$ | $e$ |
| $3$ | $-1$ |

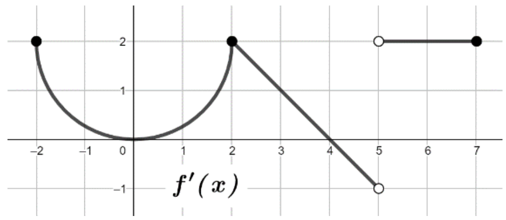
Consider $f'(x)$, the derivative of the continuous function $f$, defined on the closed interval $[-6, 7]$ except at $x = 5$. A portion of $f'$ is given in the graph above and consists of a semi circle and two line segments. The function $h(x)$ is a piecewise defined function given above where $k$ is a constant.
The function $g(x)$ and its derivatives are differentiable. Selected values for the decreasing function $g''(x)$, the second derivative of $g$, are given in the table above.

(A) Find the value of $k$ such that $h(x)$ is continuous at $x = 3$. Show your work.

(B) Using the value of $k$ found in part (A), is $h(x)$ continuous at $x = 1$? Justify your answer.

(C) Is there a time $c, -4 < c < 3$ such that $g'''(c) = -2$? Give a reason for your answer.

(D) For each $x = 2$ and $x = 4$, determine if $f(x)$ has a local minimum, local maximum or neither.
Give a reason for your answer.

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(E) Find all $x$ value(s) on the open interval $(-2, 5)$ where $f(x)$ has a point of inflection. Give a reason for your answer.

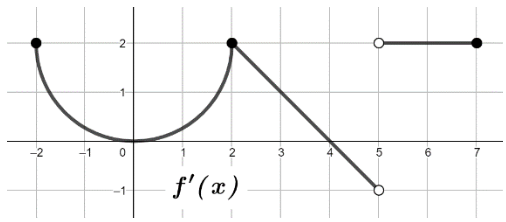
(F) Find the average rate of change of $h(x)$, in terms of $k$, over the interval $[2,5]$.

(G) If $f(3) = 5$, write an equation of the tangent line to $f(x)$ at $x = 3$.

(H) Use a right Riemann sum with the four subintervals indicated in the table to approximate $\int_{-4}^{3} g''(x)dx$.

Is this approximation an over or under estimate? Give a reason for your answer.

(I) Evaluate $\int_{0}^{7} f'(x)dx$.




$$h(x) = \begin{cases} \dfrac{sin(x-1)}{x-1}, & x < 1 \\ kx^2 - 8x + 6, & 1 \le x \le 3 \\ 4e^{2x-6} - x^2 + 5, & x > 3 \end{cases}$$

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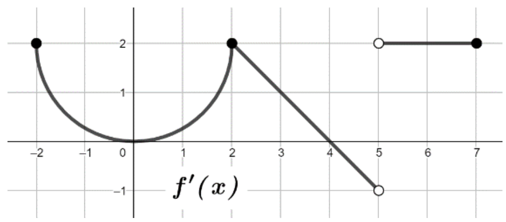
(J) Let $k(x) = x^2 + \int_1^x f'(t)dt$. Find the values for $k'(2)$ and $k''(2)$ or state that it does not exist.

(K) Find $h'(4)$.

(L) Let $m(x) = f'(x)g'\left(\frac{x}{2}\right)$. Find $m'(6)$.

(M) Let $p(x) = f(x^2 - 1)$. Find $p'(2)$.

(N) Find the average value of $f'(x)$ over the interval [2,5].

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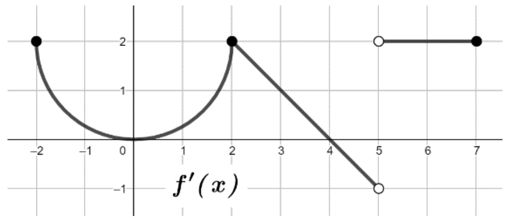
(O) Evaluate $\int_{-1}^{3} [2g'''(x) + 7]dx$

(P) If $\int_{-6}^{2} f'(x)dx = 5 - 2\pi$, then find $\int_{-2}^{-6} f'(x)dx$.

(Q) For $0 \le t \le 2.5$, a particle is moving along a horizontal axis with velocity $v(t) = \ln(g''(t))$. Is the particle speeding up or slowing down at time $t = 2$? Give a reason for your answer.

(R) Let $x$ be the number of people, in thousands, inside an amusement park. The number of people inside the park that have contracted a virus can be modeled by $v(x) = \dfrac{h(x)}{3x}$ for $3 < x < 5$. The number of people in the park is increasing at a constant rate of 0.2 thousands of people per minute. Using this model, what is the rate that people inside the park are contracting the virus with respect to time when there are four thousand people in the park?




$$h(x) = \begin{cases} \dfrac{sin(x-1)}{x-1}, & x < 1 \\ kx^2 - 8x + 6, & 1 \le x \le 3 \\ 4e^{2x-6} - x^2 + 5, & x > 3 \end{cases}$$

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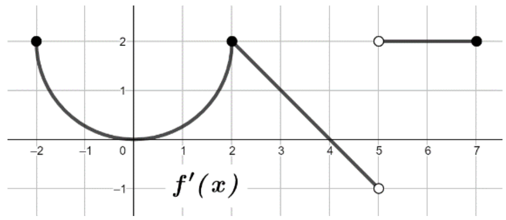
(S) $\lim\limits_{x \to 2} \dfrac{\int_4^x f'(t)dt + x}{\sin(x^2 - 4)}$

(T) Let $k = 0$, evaluate $\displaystyle\int_2^4 h(x)dx$.

(U) Is there a time $c, -4 < c < 3$, such that $g''(c) = 0$? Give a reason for your answer.

(V) Estimate $g'''(-2)$. Show the calculations that lead to your answer.

(W) For $-6 \le x \le -2, f'(x) = \dfrac{1}{4}(x+4)^3$. If $f(-2) = 0$, find the minimum value of $f(x)$ on $[-6,2]$.

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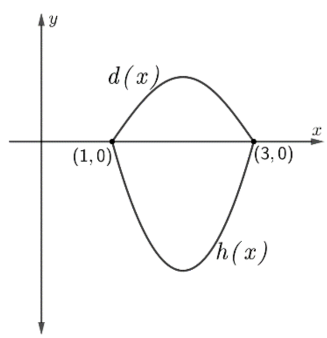
(X) Let $y = r(x)$ be the particular solution to the differential equation $\dfrac{dy}{dx} = \dfrac{h(x) + x^2}{y}$ for $x > 3$.
Find the particular solution $y = r(x)$ given the initial condition $(4, -2e)$.




(Y) The graphs of $d(x) = -\sin\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right)$ and $h(x)$ are shown above for $1 \le x \le 3$ when $k = 2$.
Find the area bounded by the graphs of $d(x)$ and $h(x)$.

(Z) Set up, but do not evaluate, an expression involving one or more integrals that gives the volume when the region bounded by the graphs above is revolved about the line $y = -5$.