Algebra 2 **Days 6 - 10**

Mrs. McCauley **Packet Work**

The unit we are studying addresses Graphs of Polynomials. Khan Academy is a great resource to add to the materials that I send you. You can watch videos on every topic, complete practice problems, and even take a practice test. You can find this unit by searching Khan Academy Algebra 2 Polynomial Graphs or use the link below:

https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-graphs

Day 6

1. Read through the notes for Polynomials covering End Behavior, Repeated Zeros, Factoring, and How Zeros, X-Intercepts, Solutions, and Factors are all connected.

2. Complete the Algebra 2 Graph Sketching – Polynomials activity. If you are able, you can use an online graphing utility like www.desmos.com or download the free Desmos app.

Day 7

1. Complete the Polynomial Sort and Match Activity. You will match polynomial graphs with the correct equation, zeros, and description cards. You will record your matches on the provided chart and return that to me for a grade.

2. Complete the Polynomial Equations Exit Ticket. You may continue to use a graphing tool like www.desmos.com.

Day 8

1. Complete the Graphs of Polynomials (factored) Quiz. Be sure to read the directions and answer every question. Do not leave anything blank. You may use www.desmos.com.

Day 9

1. Complete the 2 practice worksheets for Dividing Polynomials. This is a review from a previous lesson. You can review your notes on long division and synthetic division if needed.

Day 10

1. Read through my notes and examples for The Remainder Theorem and The Factor Theorem.

2. Complete worksheet 4-3 The Remainder and Factor Theorems. Be sure to show your work for the dividing. Use extra paper if needed.

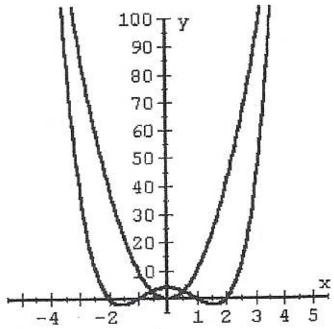
Day 6 Notes

Algebra 2 Polynomials

Studying End Behavior

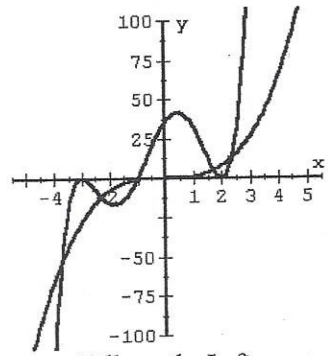
Polynomials : $f(x) = a_n x^n + \ldots + a_2 x^2 + a_1 x + a_0$

Even Degree

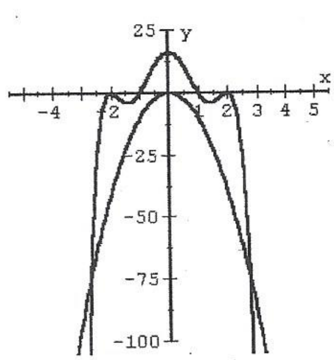


Rises on the Left
Rises on the Right
$f(x) = x^4 - 5x^2 + 4$
$a_n > 0$

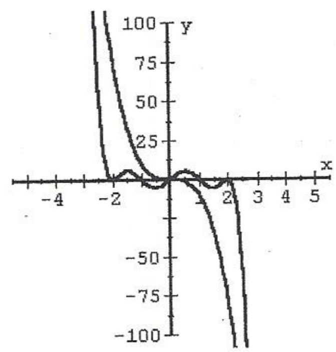
Odd Degree



Falls on the Left
Rises on the Right
$f(x) = x^5 + 3x^4 - 9x^3 - 23x^2 + 24x + 36$
$a_n > 0$



Falls on the Left
Falls on the Right
$f(x) = -x^6 + 9x^4 - 24x^2 + 16$
$a_n < 0$



Rises on the Left
Falls on the Right
$f(x) = -x^7 + 9x^5 - 24x^3 + 16x$
$a_n < 0$

Note: If the Graph Falls on the Right, then $a_n < 0$

Repeated Zeros of a Polynomial

Example 1

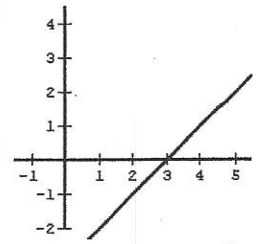
$f(x) = (x - 3)^1 = 0$

$x - 3 = 0$

$x = 3$

$x = 3$ is a zero of multiplicity 1

We say $x = 3$ is a non-repeated zero



$f(x) = (x - 3)^1$

Note: A (Real) **zero of multiplicity 1** : Crosses the x-axis rather steeply like a line.

Definition: If the factor $(x - r)$ occurs more than once in the factorization of f, then r is called a **repeated zero**, or **multiple zero** of f.

Example 2

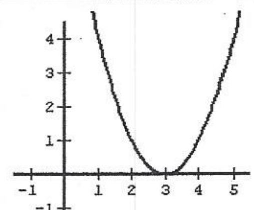
$f(x) = x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0$

$(x - 3)(x - 3) = 0$

$x = 3$ or $x = 3$

$x = 3$ is a zero of multiplicity 2



$f(x) = (x - 3)^2$

Note: A (Real) **zero of multiplicity 2** : Just touches the x-axis but does not cross the x-axis

Gets "flat" near the x-axis

"Looks" like $y = \pm x^2$ at the x-intercept

Example 3

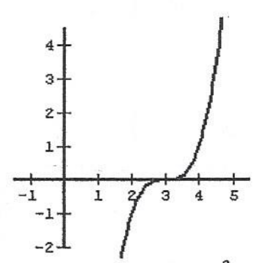
$f(x) = x^3 - 9x^2 + 27x - 27 = 0$

$(x - 3)^3 = 0$

$(x - 3)(x - 3)(x - 3) = 0$

$x = 3$ or $x = 3$ or $x = 3$

$x = 3$ is a zero of multiplicity 3



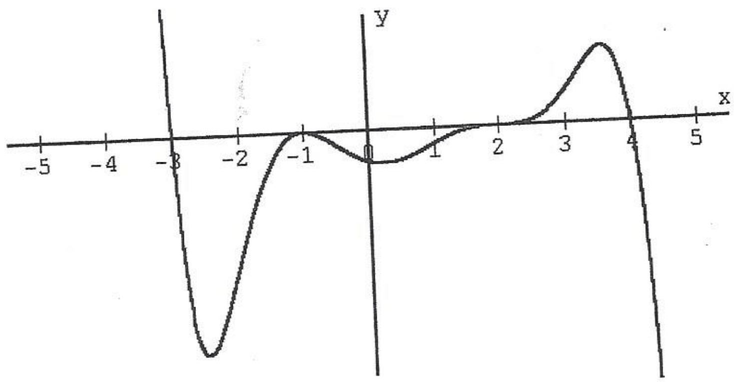
$f(x) = (x - 3)^3$

Note: A (Real) **zero of multiplicity 3** : Crosses the x-axis

Gets "flat" near the x-axis

"Looks" like $y = \pm x^3$ at the x-intercept

FACTORING A POLYNOMIAL USING ITS GRAPH



X-intercepts: $x = -3$ $\quad x = -1$ $\quad x = 2$ $\quad x = 4$

$(x+3) = 0 \quad (x+1) = 0 \quad (x-2) = 0 \quad (x-4) = 0$

Multiplicity of Zeros: 1, 2, 3, 1

Factors: $(x+3)^1 \quad (x+1)^2 \quad (x-2)^3 \quad (x-4)^1$

Polynomial: $f(x) = -(x+3)(x+1)^2(x-2)^3(x-4)$

$f(x) = -x^7 + 5x^6 + 7x^5 - 57x^4 + 26x^3 + 124x^2 - 56x - 96$

Facts:

$f(x) = a_n x^n + \ldots + a_1 x + a_0$

A polynomial of degree **n** can have :

A) At Most **n** zeros
B) At Most **n** factors
C) At Most **n** x-intercepts
D) At Most **(n – 1)** turning points (humps)

A polynomial of degree 7 can have :

A) At Most 7 zeros
B) At Most 7 factors
C) At Most 7 x-intercepts
D) At Most 6 turning points (humps)

FUNCTIONS: Zeros ↔ X-Intercepts ↔ Solutions ↔ Factors

FUNCTION: Let $f(x)$ be a polynomial function.

Example

Let $f(x) = x^2 - 2x - 3$

1) **Zero(s) of a function**

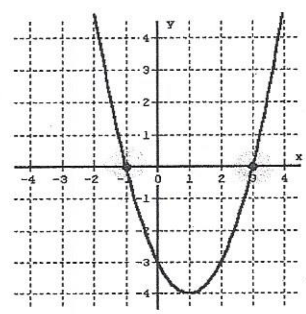
If $f(c) = \mathbf{0}$,
Then $x = c$ is a **Zero** of $f(x)$

X	Y
-2	5
-1	**0**
0	-3
1	-4
2	-3
3	**0**
4	5

$f(-1) = (-1)^2 - 2(-1) - 3 = \mathbf{0}$
$\Rightarrow x = -1$ is a **Zero** of $f(x)$

$f(3) = (3)^2 - 2(3) - 3 = \mathbf{0}$
$\Rightarrow x = 3$ is a **Zero** of $f(x)$

2) **X-Intercept(s) of the graph of a function**
$x = c$ is an **X-Intercept** of the graph of $y = f(x)$
[Note: This is only true for REAL Zeros of $f(x)$]



3) **Solve the equation**: $f(x) = 0$
$x = c$ is a **Solution** of the Equation $f(x) = 0$

$$f(x) = 0$$
$$x^2 - 2x - 3 = 0$$
$$(x + 1)(x - 3) = 0$$
$$x + 1 = 0 \text{ or } x - 3 = 0$$
$$x = -1 \text{ or } x = 3$$

$\Rightarrow x = -1$ is a **Solution**
$\Rightarrow x = 3$ is a **Solution**

4) **Factor(s) of a polynomial**
$(x - c)$ is a **Factor** of $f(x)$

$f(x) = x^2 - 2x - 3$
$= (x + 1)(x - 3)$

$\Rightarrow$ is a **Factor** of $f(x)$
$\Rightarrow (x - 3)$ is a **Factor** of $f(x)$

Day 6 Packet Work

Algebra II name: ____________________

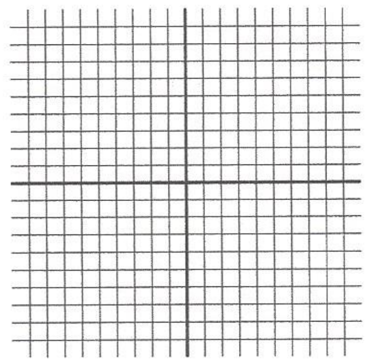
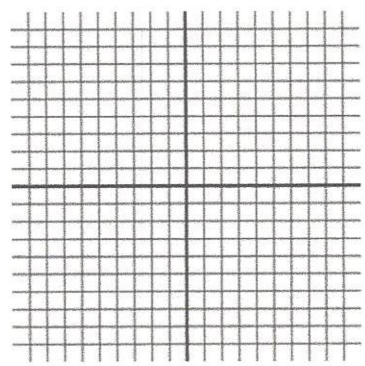
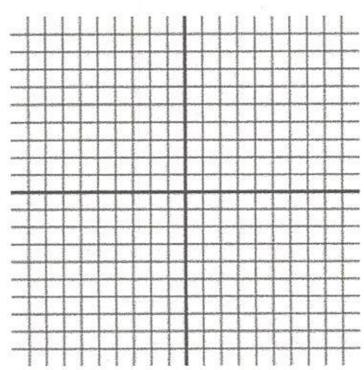
Graph Sketching – Polynomials

a. State the degree of the function and the end behavior it will exhibit.

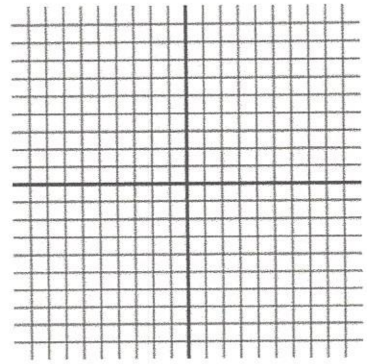
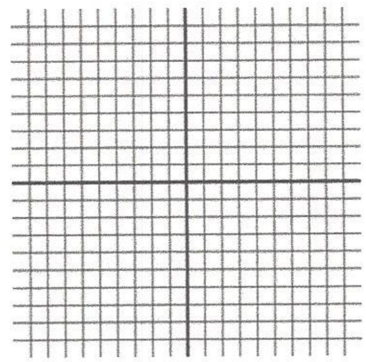
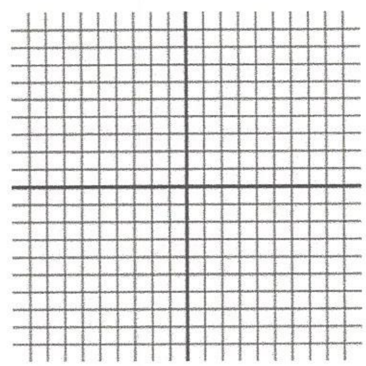
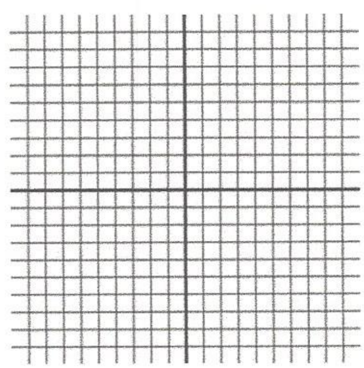
b. Identify the x – intercepts and determine if the function will cross or touch the x – axis at each x – intercept.

c. Identify the y – intercept.

d. Sketch the function on the graph provided.

1. $y = -x(x-4)(x+3)$ a. b. c.	
2. $y = -1(x+2)(x-4)^2$ a. b. c.	
3. $y = 0.5(x+2)(x-1)^3(x+4)$ a. b. c.	

Day 6 Packet Work

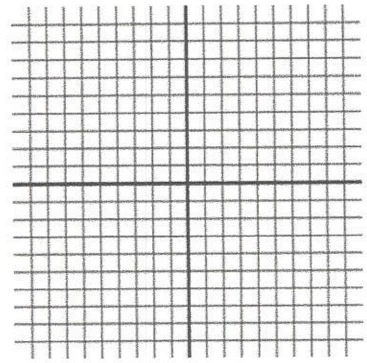
4. $y = -x^2(x-2)^3$ a. b. c.	
5. $y = (x-1)^3(x+2)^2$ a. b. c.	
6. $y = -x^3 + 2x^2 - x$ a. b. c.	
7. $y = x^3 - x^2 - 4x + 4$ a. b. c.	

8. $y = x^5 - 3x^4 - x^3 + 3x^2$

a.

b.

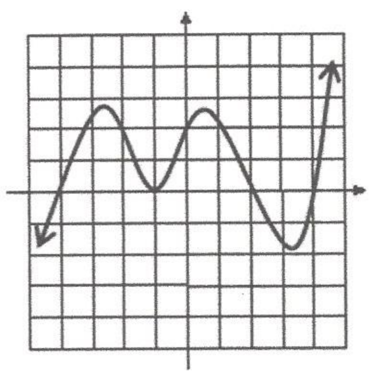
c.



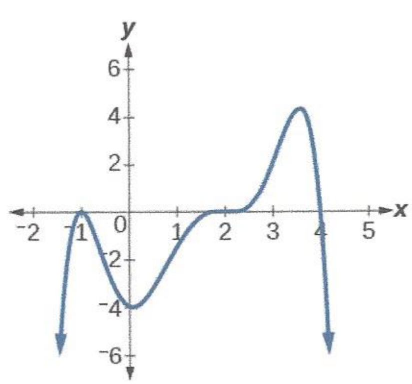
Write an equation in **factored form** for each graph shown.

CHALLENGE: Write the precise equation in **standard form** (check the $y-$*intercept*).

9.



10.



Day 7 Packet Work

Polynomial Sort and Match Activity

Name ______________________________

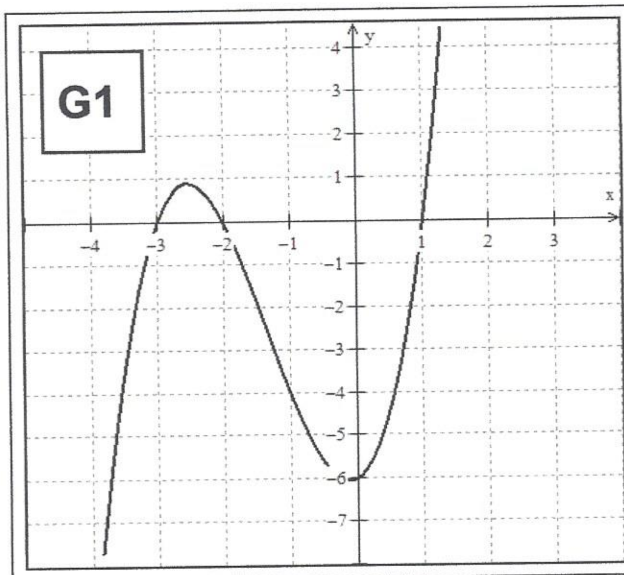
Date ________________ Period __________ Score __________

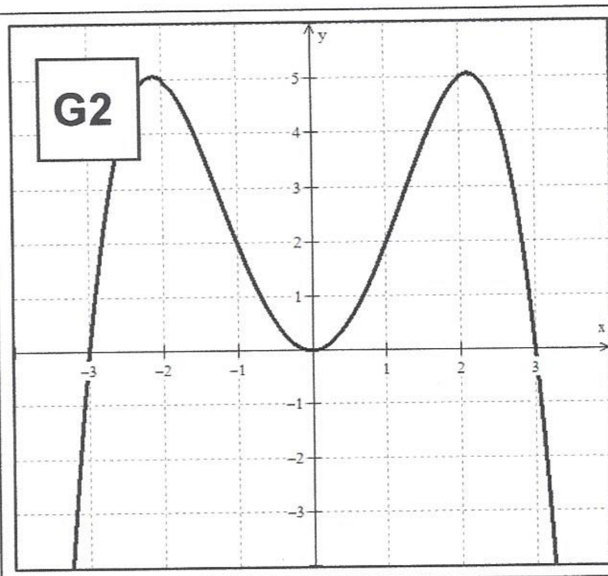
GRAPH	EQUATION	ZEROS	DESCRIPTION
G1			
G2			
G3			
G4			
G5			
G6			
G7			
G8			
G9			
G10			

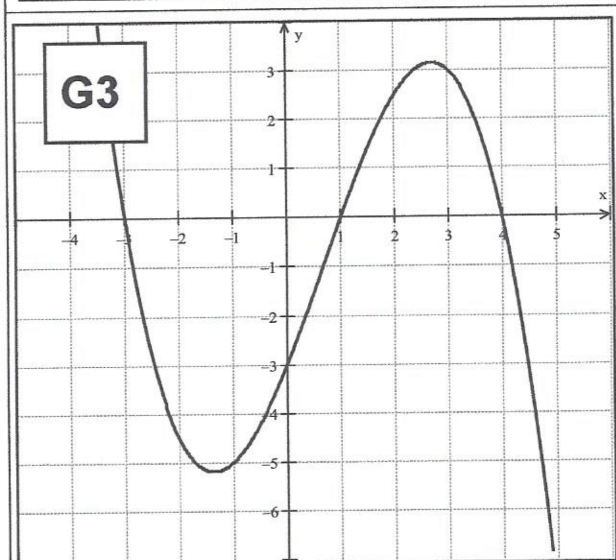
Z1 $\{-3, 1, 4\}$	Z2 $\{-2, 0, 2\}$
Z3 $\{-2, 0, 1, 2\}$	Z4 $\{-2, 1, 2, 3\}$
Z5 $\left\{-1, -2i, 2i, \frac{1}{2} + \frac{i\sqrt{3}}{2}, \frac{1}{2} - \frac{i\sqrt{3}}{2}\right\}$	Z6 $\{-3, 0, 3\}$
Z7 $\{-3, -2, 1\}$	Z8 $\{-4, -\sqrt{3}, \sqrt{3}\}$
Z9 $\{-\sqrt{3}, \sqrt{3}, 4\}$	Z10 $\{-\sqrt{3}, -1, 1, \sqrt{3}\}$

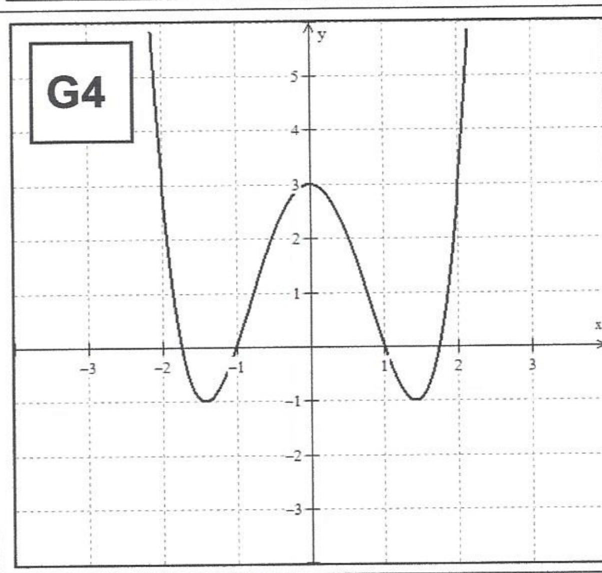
E1 $y = \frac{1}{2}(x^2 - 4)(x^2 - 4x + 3)$	E2 $y = -\frac{1}{4}(x - 4)(x^2 + 2x - 3)$
E3 $y = \frac{1}{2}(x^2 - 3)(x - 4)$	E4 $y = \frac{1}{2}(x^3 + 1)(x^2 + 4)$
E5 $y = x^3 + 4x^2 + x - 6$	E6 $y = x^5 - x^4 - 4x^3 + 4x^2$
E7 $y = \frac{1}{2}(x^2 - 3)(x + 4)$	E8 $y = x^4 - 4x^2 + 3$
E9 $y = -\frac{1}{4}x^2(x^2 - 9)$	E10 $y = -x^5 + 4x^3$

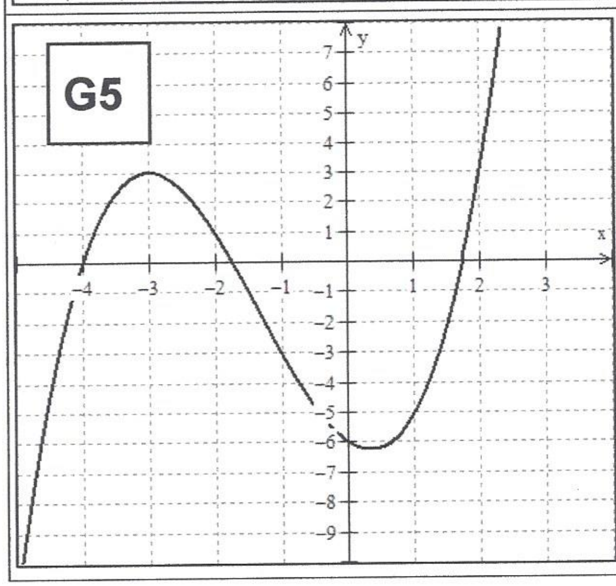
D1 This quintic polynomial has a zero at the origin with multiplicity 3.	D2 The end behavior of this quartic polynomial is up and the *y*-intercept is (0, 3).
D3 This cubic polynomial has a leading coefficient of 1 and a *y*-intercept at $(0, -6)$.	D4 This fifth degree polynomial has 5 rational roots. The zero at the origin has multiplicity 2.
D5 The end behavior of this cubic polynomial is up on the left and down on the right.	D6 The *y*-intercept is (0, 6). The leading coefficient is $\frac{1}{2}$.
D7 The end behavior of this quartic polynomial is down and down since the leading coefficient is negative.	D8 This fifth degree polynomial has 1 real root and 4 complex roots.
D9 This cubic polynomial has two irrational roots, one rational root, and a *y*-intercept at $(0, -6)$.	D10 This quartic polynomial does not have y-axis symmetry.

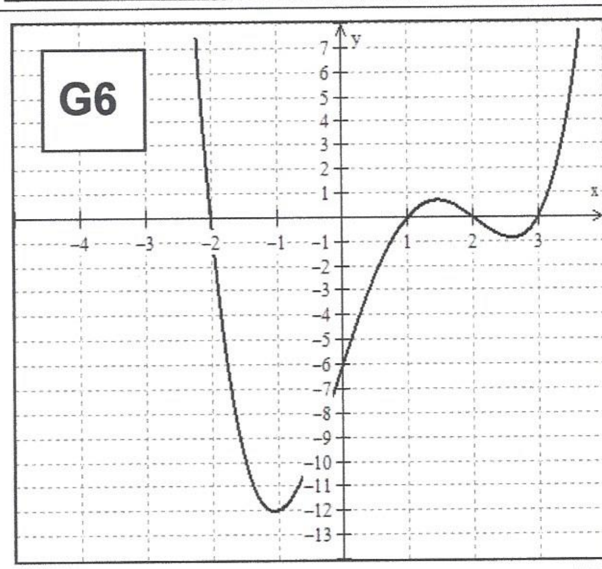

G1
x
y

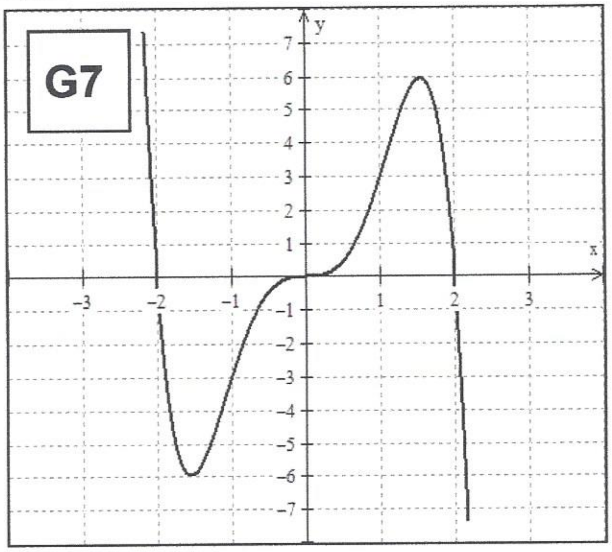

G2
x
y

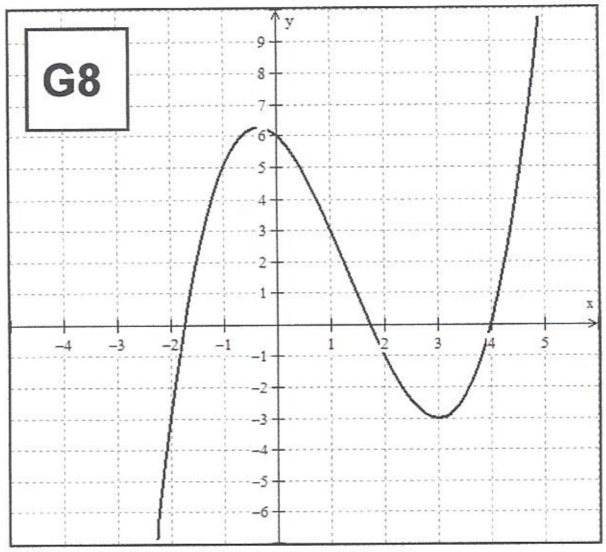

G3
x
y

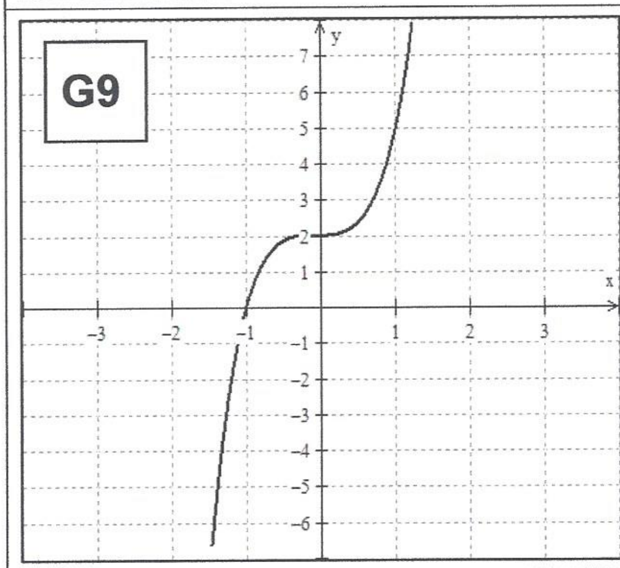

G4
x
y

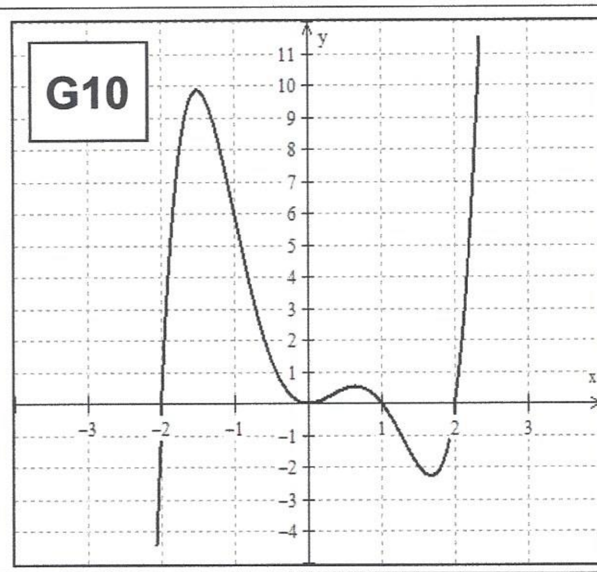

G5
x
y


G6
x
y









 Jean Adams

Day 7 Packet Work

Algebra II

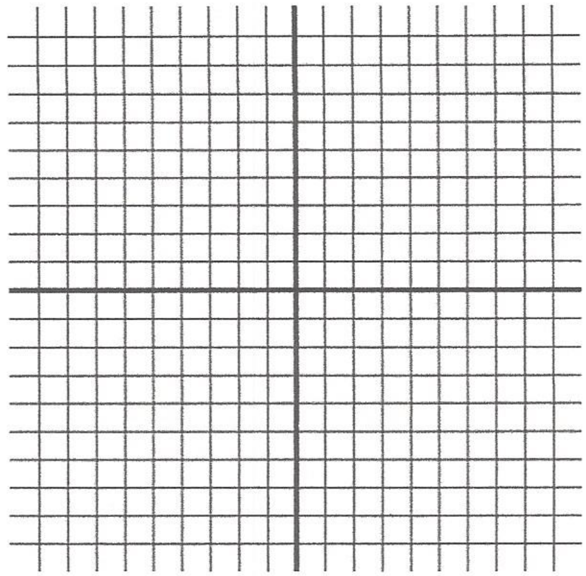
Polynomial Equations Exit Ticket name:

1. $y = x(x-2)(x+5)$

end behavior:

x-intercepts & cross or touch?

y-intercept:

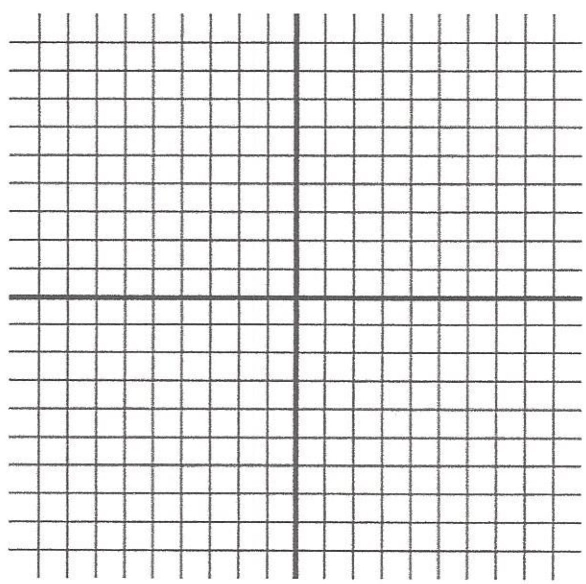


2. $f(x) = x^4 + 6x^3 - 22x + 15$

end behavior:

x-intercepts & cross or touch?

y-intercept:



Day 8 Packet Work

name: ______________________________

Quiz Graphs of Polynomials (Factored)

Directions:

For each polynomial, identify the degree, end behavior, x – intercepts, behavior at each x – intercept, y – intercept, and sketch the graph of the function. Record your answers on the answer sheet provided.

1. $y = x(x-1)(x+3)$

2. $y = -(x+3)(x-2)^2$

3. $y = x^2(x-4)^2$

4. $y = -x(x+1)^2(x-2)$

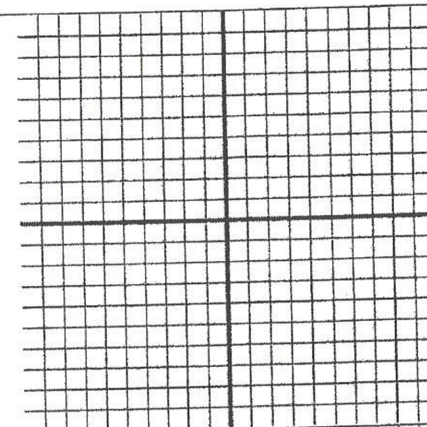
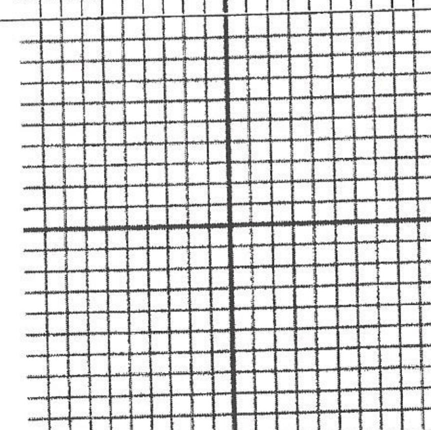
5. $y = (x-3)^2(x+1)^3$

Day 8

Answer Sheet

name: ____________________

QUIZ – Graphing Polynomials (Factored)

	degree	end behavior (left & right?)	x – intercepts & behavior (cross or touch?)	y - intercept	sketch the graph
1.					
2.					

3.					
4.					
5.					

Day 9 Packet work

name:

Practice – Algebra 2

Dividing Polynomials

Show your work!

1. $\frac{27x^3-9x^2+3x}{3x}$

2. $\frac{x^4+3x^2-x^2-x+6}{x+3}$

3. $\frac{xx^4+8x^3-5x^2-4x+2}{x^2+4x-2}$

4. $(2x^4 + 7x^3 - 7x^2 - 16x + 11) \div (2x - 3)$

5. $(10x^3 - x^2 + 8) \div (x - 1)$

NAME ______________________ DATE Day 9

Student Edition
Pages 675–680

Practice

Show your work!

Dividing Polynomials

Find each quotient.

1. $(x^2 - 3x - 40) \div (x + 5)$

2. $(n^2 - 13n + 36) \div (n - 9)$

3. $(c^2 + 5c + 20) \div (c - 3)$

4. $(y^2 - 3y - 2) \div (y + 7)$

5. $(6r^2 - 5r - 56) \div (3r + 8)$

6. $(20w^2 + 39w + 18) \div (5w + 6)$

7. $\dfrac{x^3 + 6x^2 + 3x + 1}{x - 2}$

8. $\dfrac{x^3 - 3x^2 - 6x - 20}{x - 5}$

9. $\dfrac{z^3 - 4z^2 + z + 6}{z - 2}$

10. $\dfrac{6d^3 + d^2 - 2d + 17}{2d + 3}$

Day 10 Notes & Examples

Remainder Theorem	Factor Theorem
If the polynomial $f(x)$ is divided by $(x-c)$, then the remainder is $f(c)$.	Let $f(x)$ be a polynomial. If $f(c)=0$, then $(x-c)$ is a factor of $f(x)$. If $(x-c)$ is a factor of $f(x)$, then $f(c)=0$. If $(x-c)$ is a factor of $f(x)$ or if $f(c)=0$, then c is called a zero of $f(x)$.

Example $f(x)=3x^3+4x^2-5x+7$. Find $f(-4)$ using

use synthetic substitution:

$f(-4) = -101$

-4	3	4	-5	7
	↓	-12	32	-108
	3	-8	27	(-101) ← remainder is answer

Example Solve the equation $2x^3-3x^2-11x+6=0$ given that -2 is a zero of $f(x)=2x^3-3x^2-11x+6$.

$x=-2$
$x=\frac{1}{2}$
$x=3$

① use synthetic substitution:

-2	2	-3	-11	6
	↓	-4	14	-6
	2	-7	3	0 ← remainder of zero means (-2 is a zero)

② Put variables back on answer

$2x^2-7x+3$

③ Factor $2x^2-7x+3$

$(2x-1)(x-3)$

④ set factors = 0 and solve

$2x-1=0$ $\quad x-3=0$

$2x=1$ $\quad (x=3)$

$(x=\frac{1}{2})$

Example Use synthetic division and the Remainder Theorem to find the indicated function value

$$f(x) = x^3 - 7x^2 + 5x - 6;\ f(3)$$

3	1	-7	5	-6
	↓	3	-12	-21
	1	-4	-7	-27

$f(3) = -27$

Example Is $(x + 1)$ a factor of $f(x) = x^3 - 4x^2 + x + 6$? How do you know?

① set $x+1=0$ and solve
$x=-1$

② use synthetic substitution

-1	1	-4	1	6
	↓	-1	5	-6
	1	-5	6	0

yes, because the remainder was zero!

Example Find all the zeros of $f(x) = 2x^3 - 5x^2 + x + 2$ given that 2 is zero.

① Divide

2	2	-5	1	2
	↓	4	-2	-2
	2	-1	-1	0

② Factor $2x^2 - 1x - 1$
$(2x+1)(x-1)$

③ Solve for x

$2x+1=0$ | $x-1=0$
$2x=-1$ | $x=1$
$x=-\frac{1}{2}$

Answers: $x=2$ $x=-\frac{1}{2}$ $x=1$

Example Find all the zeros of $f(x) = 12x^3 + 16x^2 - 5x - 3$ given that $-\frac{3}{2}$ is zero.

① Divide

$-\frac{3}{2}$	12	16	-5	-3
	↓	-18	3	3
	12	-2	-2	0

② Factor $12x^2-2x-2$
$2(6x^2-x-1)$
$2(3x+1)(2x-1)$

③ Solve for x

$3x+1=0$ | $2x-1=0$
$3x=-1$ | $2x=1$
$x=-\frac{1}{3}$ | $x=\frac{1}{2}$

Answers: $x=\frac{-3}{2}$ $x=\frac{-1}{3}$ $x=\frac{1}{2}$

Day 10

NAME ______________________ DATE Packet Work

4-3

Practice Worksheet

The Remainder and Factor Theorems

Divide using synthetic division.

1. $(x^2 - 5x - 12) \div (x + 3)$

2. $(3x^2 + 4x - 12) \div (x - 5)$

3. $(2x^3 + 3x^2 - 8x + 3) \div (x + 3)$

4. $(x^4 - 3x^2 + 1) \div (x - 1)$

Find the remainder for each division. Is the divisor a factor of the polynomial?

5. $(2x^3 - 3x^2 - 10x + 3) \div (x - 3)$

6. $(2x^4 + 4x^3 - x^2 + 9) \div (x + 2)$

7. $(10x^3 - x^2 + 8x + 29) \div \left(x + \frac{2}{5}\right)$

8. $(2x^4 + 14x^3 - 2x^2 - 14x) \div (x + 7)$

Use the remainder theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

9. $(3x^3 - 2x^2 + x - 4) \div (x - 2)$

10. $(x^4 - x^3 - 10x^2 + 4x + 24) \div (x + 2)$

11. $(x^4 + 5x^3 - 14x^2) \div (x + 7)$

12. $(x^3 + x^2 - 10) \div (x + 3)$

Find the value of k so that each remainder is zero.

13. $(2x^3 + kx^2 + 7x - 3) \div (x - 3))$

14. $(x^3 + 9x^2 + kx - 12) \div (x + 4)$

15. Determine how many times 2 is a root of $x^3 - 7x^2 + 16x - 12 = 0$.