

**Rational Zeros Theorem****Lesson Notes & Examples****Algebra 2****RECALL:**

- The zeros of a polynomial are found by setting the factors of the polynomial equal to zero and solving for  $x$ .
- When a polynomial is divided by  $(x - c)$ , then the remainder is  $f(c)$ .
- If a polynomial is divided by  $(x - c)$  and the remainder or  $f(c)$  is equal to zero, then  $(x - c)$  is a factor of the polynomial.

**RATIONAL ZEROS THEOREM:**

Let  $f$  be a polynomial function of degree 1 or higher of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where each coefficient is an integer. If  $\frac{p}{q}$ , in lowest terms, is a rational zero of  $f$ , then  $p$  must be a factor of  $a_0$  and  $q$  must be a factor of  $a_n$ .

**EXAMPLES:**

1. List the potential rational zeros of  $f(x) = 2x^3 + 3x^2 - 8x + 3$ .

$$\frac{\text{factors of } 3}{\text{factors of } 2} : \pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$$

2. List the potential rational zeros of  $f(x) = x^3 + 5x^2 - 12x - 36$ .

$$\frac{\text{factors of } 36}{\text{factors of } 1} : \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \\ \pm 9, \pm 12, \pm 16, \pm 36$$

HOW TO FIND THE REAL ZEROS OF A POLYNOMIAL:

1. Always try to factor the polynomial first. If the polynomial is factorable, then set each factor equal to zero and solve for  $x$  to find the  $x$ -intercepts.
2. If you cannot factor, make a list of the potential rational zeros,  $\frac{p}{q}$ 's.
3. Use synthetic division to check all your potential rational zeros until you find a factor (get a remainder equal to zero).
4. After you find a factor, try to factor the depressed polynomial. If it will not factor, go back to your list of potential rational zeros and keep dividing. Remember that polynomials can have double or more roots, so if you find a factor, try to divide by that number again. The depressed polynomial may still be divisible by that factor. You could also try the quadratic formula instead of factoring once your degree is quadratic.

EXAMPLES:

Find the real zeros of the function. Write  $f$  in factored form.

3.  $f(x) = 2x^3 + 3x^2 - 8x + 3$

possible rational zeros:  $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}$

Use synthetic division to check each possible zero. Remember, you are trying to get a remainder of zero.

$$\begin{array}{r|rrrr} -1 & 2 & 3 & -8 & 3 \\ & \downarrow & & & \\ \hline & 2 & 1 & -9 & 12 \end{array}$$

$(x+1)$  is not a factor!  
NO! ☹

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -8 & 3 \\ & \downarrow & & & \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$(x-1)$  is a factor!  
YES! ☺

$2x^2 + 5x - 3$

Now try to factor the depressed polynomial.

$$2x^2 + 5x - 3$$

$$(2x - 1)(x + 3)$$

factored form:  
 $(x-1)(2x-1)(x+3)$

$$4. f(x) = x^4 - x^3 + x^2 - 3x - 6$$

Possible Zeros:  $\pm 1, \pm 2, \pm 3, \pm 6$

Now Divide.

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & 1 & -3 & -6 \\ & \downarrow & -1 & 2 & -3 & 6 \\ \hline & 1 & -2 & 3 & -6 & 0 \end{array} \text{ YAY!}$$

$(x+1)$  is a factor!

factored form:

$$(x+1)(x^2+3)(x-2)$$

depressed polynomial

$$x^3 - 2x^2 + 3x - 6$$

$$(x^3 - 2x^2) + 3x - 6$$

factor it!

$$x^2(x-2) + 3(x-2)$$

$$(x^2+3)(x-2)$$

$$5. f(x) = 10x^3 - 15x^2 - 16x + 12$$

possible zeros:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Oh dear! There are so many!!!

$$\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{4}{5}, \pm \frac{6}{5}, \pm \frac{12}{5}$$

$$\pm \frac{1}{10}, \pm \frac{3}{10}$$

Try to divide and find something that gives us a remainder of zero.

$$\begin{array}{r|rrrr} 1 & 10 & -15 & -16 & 12 \\ & \downarrow & 10 & -5 & -2 \\ \hline & 10 & -5 & -21 & -9 \end{array} \text{ No!}$$

$$\begin{array}{r|rrrr} 2 & 10 & -15 & -16 & 12 \\ & \downarrow & 20 & 10 & -12 \\ \hline & 10 & 5 & -6 & 0 \end{array} \text{ YAY!}$$

$(x-2)$  is a factor!

$$\begin{array}{r|rrrr} -1 & 10 & -15 & -16 & 12 \\ & \downarrow & -10 & 25 & -9 \\ \hline & 10 & -25 & 9 & 3 \end{array} \text{ No!}$$

depressed polynomial

$$10x^2 + 5x - 6$$

will not factor further

factored form:

$$(x-2)(10x^2+5x-6)$$

**Rational Zeros Theorem****Practice Problems****Algebra 2**

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List all the possible rational zeros of each function. Use synthetic division and factoring to find the rational zeros. Write your function in factored form.

1.  $f(x) = x^3 + 3x^2 - 6x - 8$

2.  $f(x) = x^4 - 3x^3 - 11x^2 + 3x + 10$

3.  $f(x) = x^3 - x^2 - 8x + 12$

4.  $f(x) = x^3 + 4x^2 - 2x + 15$

Algebra II

name:

Graph Polynomial Equations Examples

|                                                                                                                                                                                                                                                                                                                                                         |  |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| <p>1. <math>y = (x + 3)(x - 2)^2</math><br/>YAY! Already factored!</p>                                                                                                                                                                                                                                                                                  |  |
| <p>end behavior:<br/>degree = 3 and positive<br/>fall left + rise right</p>                                                                                                                                                                                                                                                                             |  |
| <p>x-intercepts &amp; cross or touch?<br/> <math>x + 3 = 0</math>      <math>x - 2 = 0</math><br/> <math>x = -3</math>        <math>x = 2</math><br/>                     cross              touch<br/> <math>(-3, 0)</math>        <math>(2, 0)</math></p>                                                                                             |  |
| <p>y-intercept:<br/><math>(0, 12)</math></p>                                                                                                                                                                                                                                                                                                            |  |
| <p>2. <math>y = -x^2(x + 4)(x - 5)</math></p>                                                                                                                                                                                                                                                                                                           |  |
| <p>end behavior:<br/>negative + degree = 4<br/>fall left + fall right</p>                                                                                                                                                                                                                                                                               |  |
| <p>x-intercepts &amp; cross or touch?<br/> <math>x^2 = 0</math>      <math>x + 4 = 0</math>      <math>x - 5 = 0</math><br/> <math>x = 0</math>        <math>x = -4</math>        <math>x = 5</math><br/>                     touch        cross              cross<br/> <math>(0, 0)</math>        <math>(-4, 0)</math>        <math>(5, 0)</math></p> |  |
| <p>y-intercept:<br/><math>(0, 0)</math></p>                                                                                                                                                                                                                                                                                                             |  |

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |  |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|
| <p>3. <math>y = 5x^3 - x^2 - 5x + 1</math><br/>                     factor <math>\ddot{=}</math><br/> <math>x^2(5x-1) - 1(5x-1)</math><br/> <math>(5x-1)(x^2-1)</math><br/> <math>(5x-1)(x+1)(x-1)</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |
| <p>end behavior:<br/>                     odd + positive<br/>                     fall left + rise right</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |  |
| <p>x-intercepts &amp; cross or touch?<br/> <math>5x-1=0</math>    <math>x+1=0</math>    <math>x-1=0</math><br/> <math>x=1/5</math>    <math>x=-1</math>    <math>x=1</math><br/>                     cross    cross    cross</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |  |
| <p>y-intercept:<br/> <math>(0, 1)</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |  |
| <p>4. <math>y = x^4 - 3x^3 - 11x^2 + 3x + 10</math><br/>                     find the zeros <math>\ddot{=}</math><br/> <math>\begin{array}{r l} \downarrow &amp; 1 \quad -3 \quad -11 \quad 3 \quad   \quad 10 \\ &amp; \downarrow \quad 1 \quad -2 \quad -13 \quad   \quad -10 \\ \hline &amp; 1 \quad -2 \quad -13 \quad -10 \quad   \quad 0 \end{array}</math><br/> <math>x=1</math> zero<br/> <math>\begin{array}{r l} \downarrow &amp; 1 \quad -2 \quad -13 \quad   \quad -10 \\ &amp; \downarrow \quad -1 \quad 3 \quad   \quad 10 \\ \hline &amp; 1 \quad -3 \quad -10 \quad   \quad 0 \end{array}</math><br/> <math>x=-1</math> zero<br/> <math>x^2-3x-10</math><br/>                     will factor<br/> <math>(x-5)(x+2)</math></p> |  |
| <p>end behavior:<br/>                     even + positive<br/>                     rise left + rise right</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |
| <p>x-intercepts &amp; cross or touch?<br/> <math>x=1</math>    <math>x=-1</math>    <math>x-5=0</math>    <math>x+2=0</math><br/>                     cross    cross    cross    cross<br/> <math>x=5</math>    <math>x=-2</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |  |
| <p>y-intercept:<br/> <math>(0, 10)</math></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |  |

This is tough!  
 Watch my video!

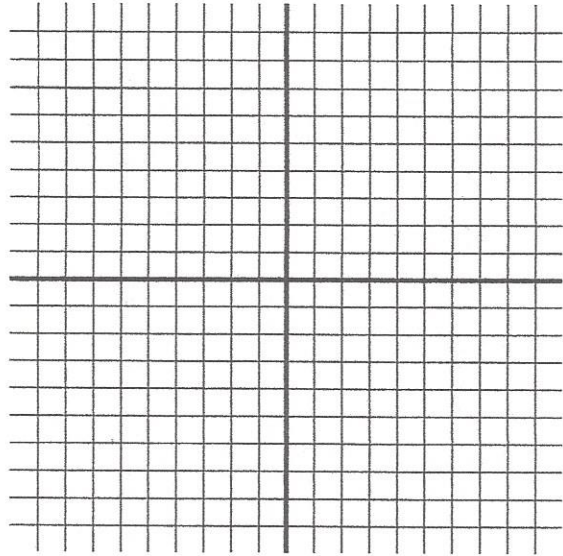
## Graphing Polynomials

Name: \_\_\_\_\_

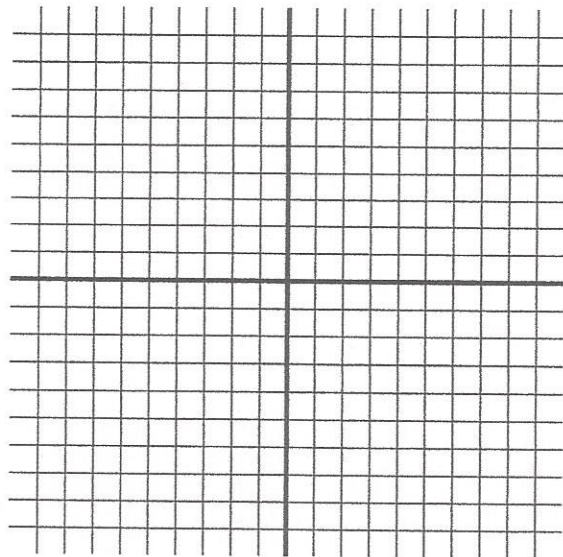
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Use what you know about polynomials to graph the following polynomial functions.

1.  $P(x) = -(x - 1)(x + 1)^2$

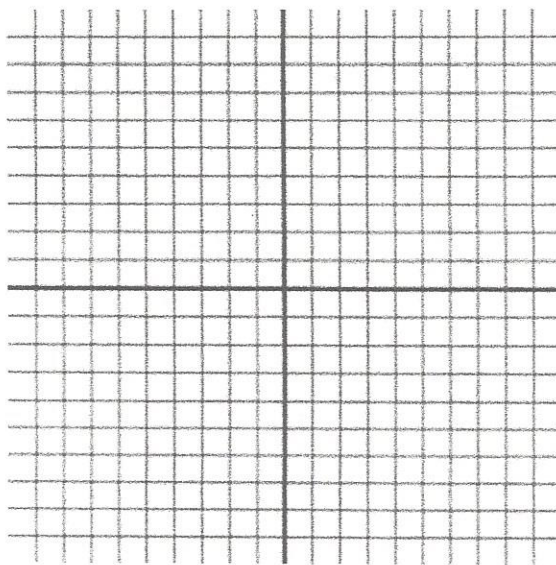


2.  $P(x) = x^3 - 6x^2 + 9x$

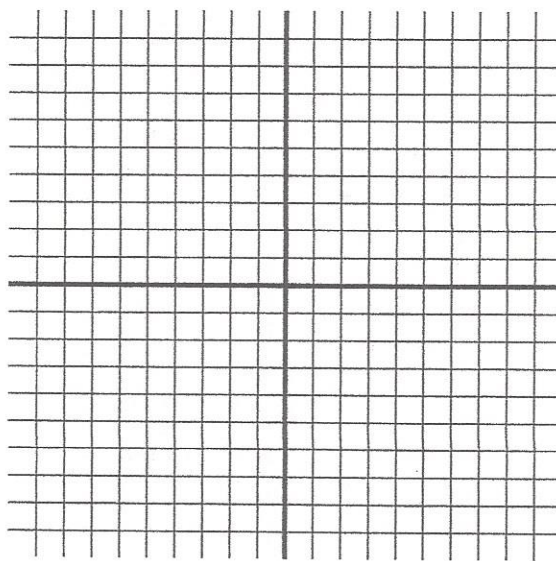




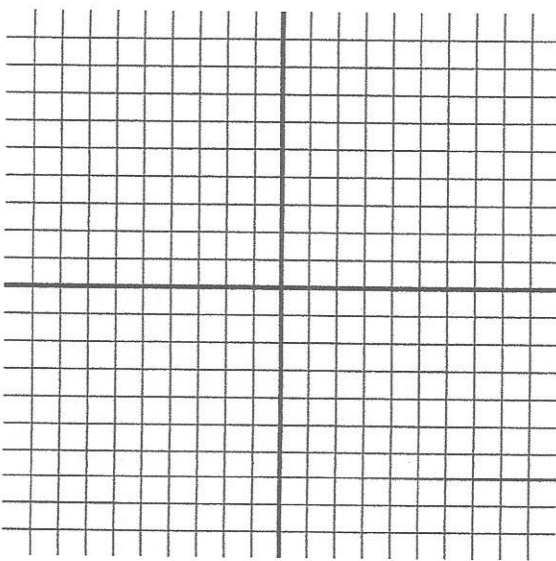
3.  $P(x) = 2x^3 - 5x^2 - 9x + 18$



4.  $P(x) = -x^3 + 7x - 6$



5.  $P(x) = x^3 + x^2 + 4x + 4$



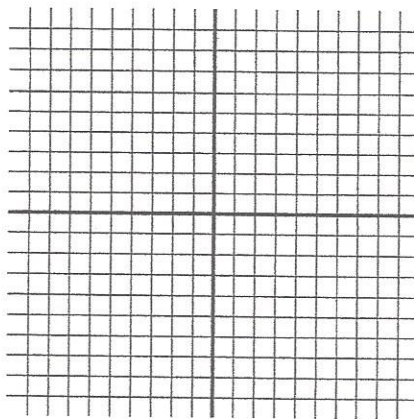
## Algebra 2

## Graphing Polynomials PRACTICE

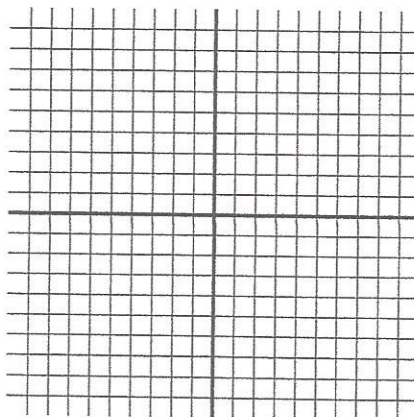
name: \_\_\_\_\_

Use what you have learned about end behavior, zeros, and  $y$ -intercepts to sketch a graph of each polynomial. Show the work that leads to your graph.

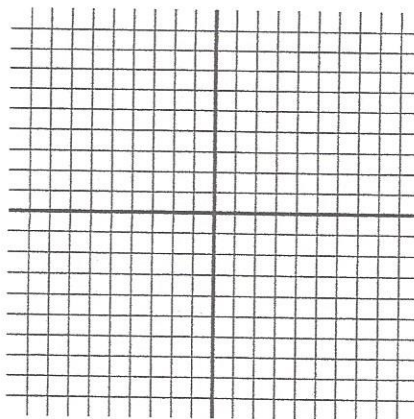
1.  $y = -x(x - 1)^2$



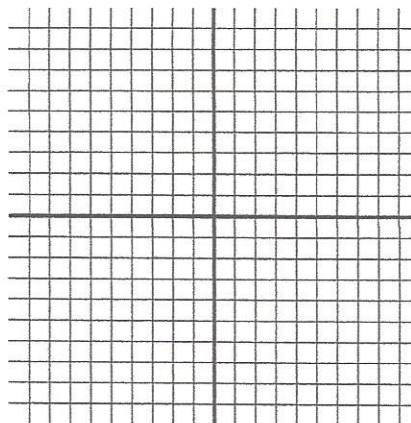
2.  $f = -(x - 1)(x + 3)(x - 4)$



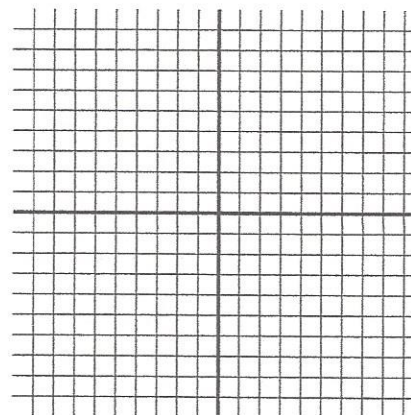
3.  $P(x) = x^4 - 10x^2 + 9$



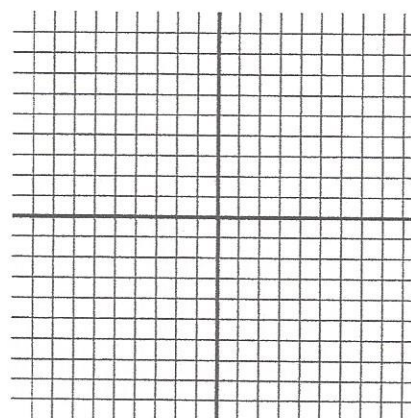
4.  $y = -x^4 - 5x^3 + 14x^2$



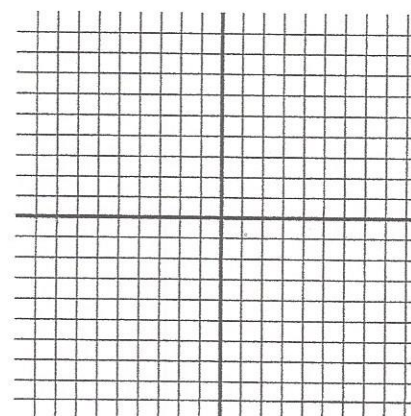
5.  $P(x) = x^3 - 9x^2 + 27x - 27$



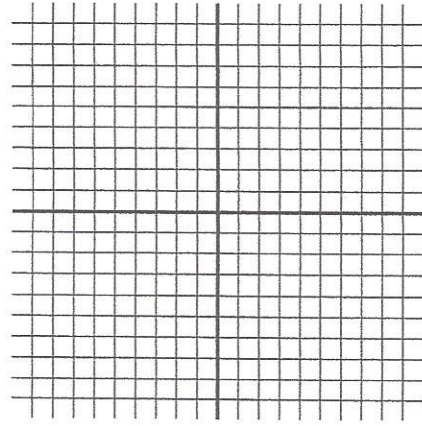
6.  $y = x^4 - 4x^3 - 7x^2 + 34x - 24$



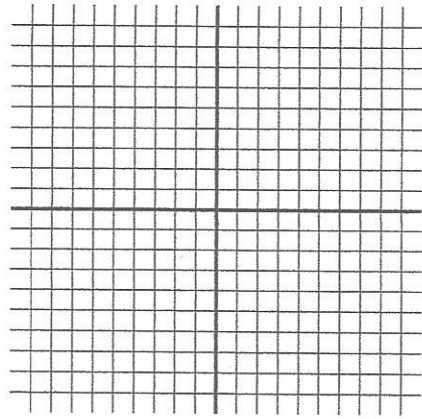
7.  $f(x) = x^3 + 6x^2 + 3x - 10$



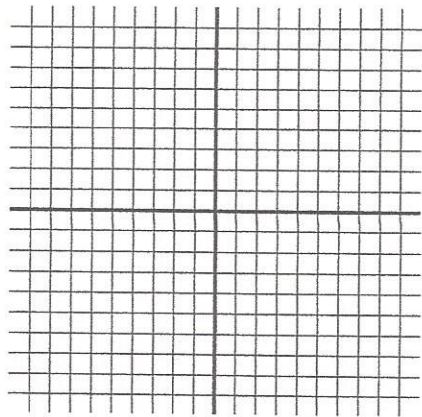
8.  $P(x) = x^4 - 2x^3 - 4x^2 + 11x - 6$



9.  $y = x^3 - 7x^2 + 16x - 12$

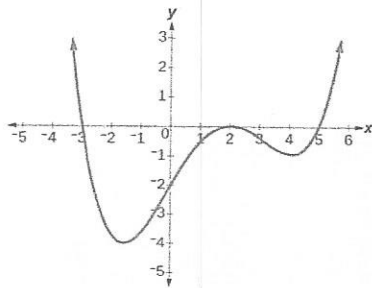


10.  $y = x^3 + 4x^2 - 2x + 15$

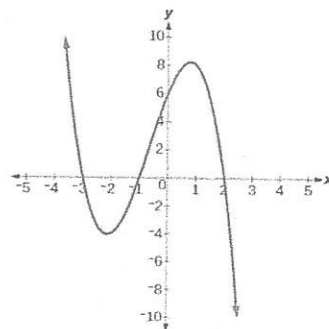


Write a possible polynomial equation in factored form for the graphs below.

11.



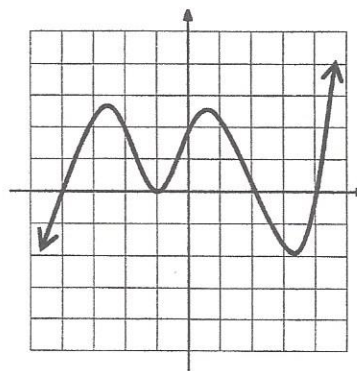
12.



**Polynomials Practice  
Putting it all together**

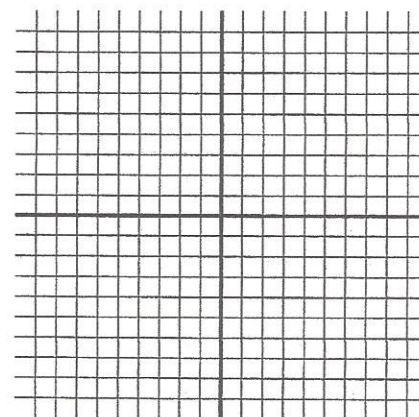
Name: \_\_\_\_\_

- 1) Write the equation for the polynomial graph shown. Assume that all even multiplicities are 2 and all odd multiplicities are 1



- 2) Sketch the graph of the equation with a double root at 0, a single root at 5, and a triple root at -2. Assume the leading coefficient is negative one. Write the equation of the function that describes the graph.

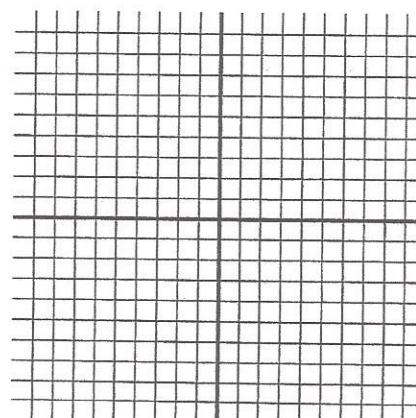
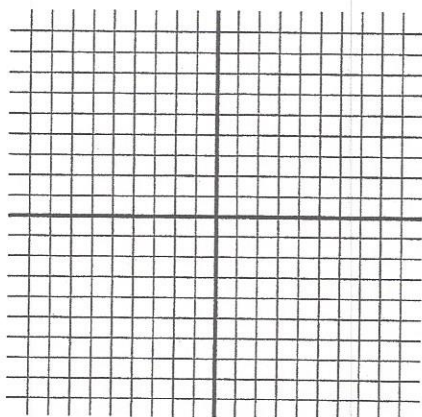
Equation: \_\_\_\_\_



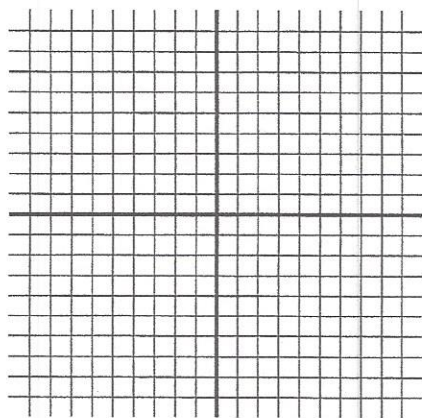
**Sketch the graph of the equation. Show the work that leads to your answer.**

3)  $y = -x(x - 4)^2$

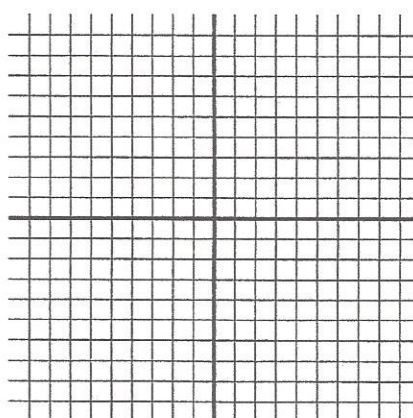
4)  $f(x) = (x + 1)(x - 3)(x + 4)$



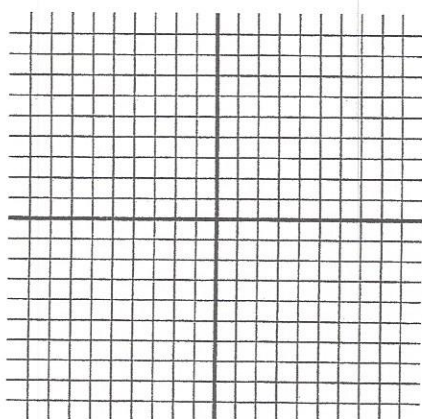
5)  $P(x) = 2x^3 + 10x^2 + 12x$



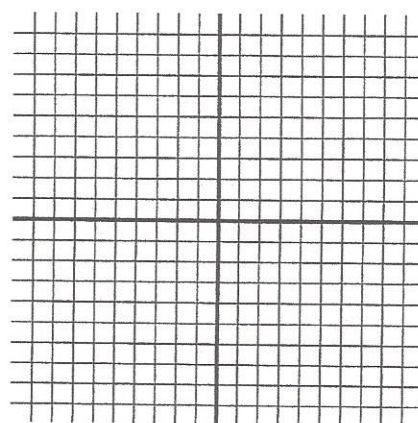
6)  $y = x^4 - 17x^2 + 16$



7)  $f(x) = x^4 - 7x^3 + 9x^2 + 11x - 6$

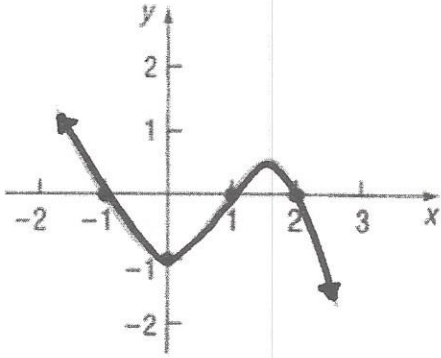


8)  $y = x^4 - 2x^3 - 3x^2 + 8x - 4$

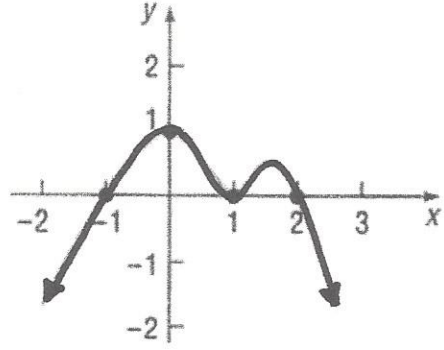


Match the graphs with the equations. How did you make your choices?

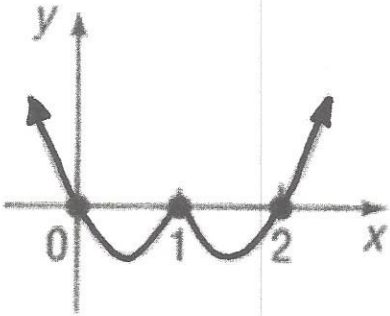
A.



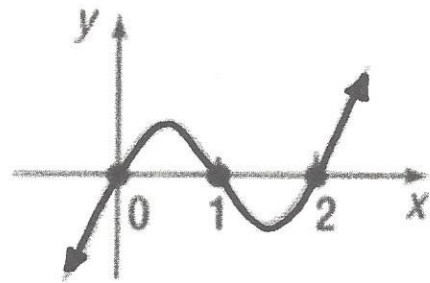
B.



C.



D.



\_\_\_\_\_ 9.  $y = x^3 - 3x^2 + 2x$

\_\_\_\_\_ 10.  $y = -\frac{1}{2}(x+1)(x-1)^2(x-2)$

\_\_\_\_\_ 11.  $y = -\frac{1}{2}(x-1)(x+1)(x-2)$

\_\_\_\_\_ 12.  $y = x^4 - 4x^3 + 5x^2 - 2x$

Algebra II

name: \_\_\_\_\_

## Polynomials STUDY GUIDE

1. Use long division

$$(2x^3 - x^2 - 13x - 6) \div (2x + 1)$$

2. Use synthetic division

$$(8x^3 + 6x^2 + 2x + 19) \div (x - 2)$$

3. Is  $(x - 2)$  a factor of the polynomial  $y = 6x^3 - x^2 + 3x - 5$ ? Why?4. Is  $(x - 3)$  a factor of the polynomial  $y = x^3 - 7x^2 + 7x + 15$ ? Why?



5. What is the end behavior of the function  $f(x) = -2x^3 - x^2 - 5x + 3$  when it is graphed?

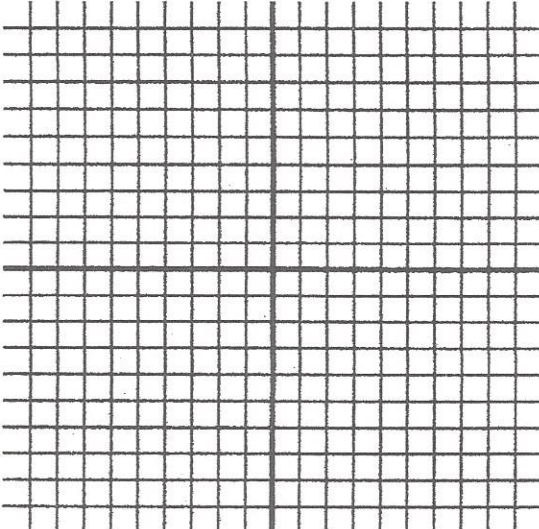
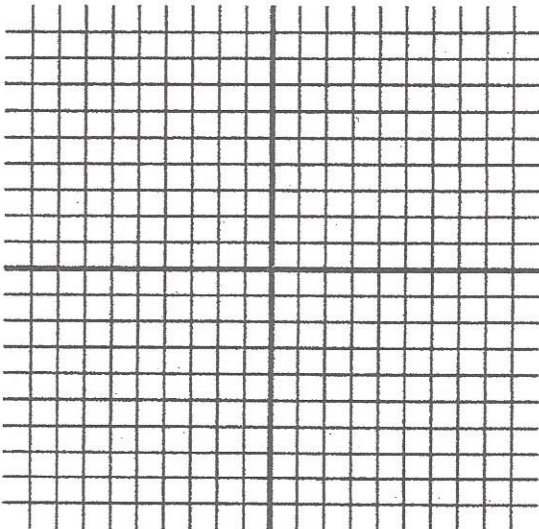
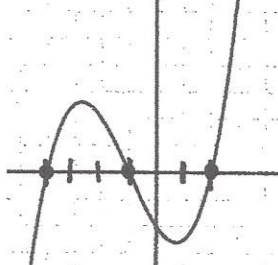
6. What is the end behavior of the function  $f(x) = x^4 - x^3 + 3x^2 - x + 1$  when it is graphed?

7. Is  $-1$  a zero of the polynomial  $y = 2x^3 + 5x^2 - 6x - 9$ ?

8. List all the possible rational roots for the polynomial

$$y = x^4 - 3x^3 - x^2 + 2x - 15.$$

For each function below: identify the key features and sketch a graph of the polynomial.

|                                                                                             |                                                                                      |
|---------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|
| <p>9. <math>y = (x + 1)(x - 2)^2(x - 4)</math></p>                                          |    |
| <p><i>end behavior:</i></p>                                                                 |                                                                                      |
| <p><i>x-intercepts &amp; cross or touch?</i></p>                                            |                                                                                      |
| <p><i>y-intercept:</i></p>                                                                  |                                                                                      |
| <p>10. <math>y = x^3 - 4x^2 - x + 4</math></p>                                              |   |
| <p><i>end behavior:</i></p>                                                                 |                                                                                      |
| <p><i>x-intercepts &amp; cross or touch?</i></p>                                            |                                                                                      |
| <p><i>y-intercept:</i></p>                                                                  |                                                                                      |
| <p><b>BONUS:</b><br/>Write a possible equation for the polynomial graphed to the right.</p> |  |

Algebra II

name: \_\_\_\_\_

Polynomials TEST

Form A

1. Use long division

$$(3x^3 + 5x^2 + 8x + 7) \div (3x + 2)$$

2. Use synthetic division

$$(3x^3 + 4x^2 - 5x - 2) \div (x - 1)$$

3. Is  $(x - 4)$  a factor of the polynomial  $= 2x^4 - 7x^3 - 8x^2 + 14x + 8$ ? Why?

4. Is  $(x - 3)$  a factor of the polynomial  $y = 3x^3 + 4x^2 - 5x - 2$ ? Why?

5. What is the end behavior of the function  $f(x) = x^3 + 4x^2 + 7x - 9$  when it is graphed?

6. What is the end behavior of the function  $f(x) = -x^4 - 2x^3 + 3x^2 + 1$  when it is graphed?

7. Is 3 a zero of the polynomial  $= x^4 - 6x^3 + 10x^2 - 6x + 9$  ?

8. List all the possible rational roots for the polynomial

$$y = x^3 - x^2 + x - 6$$

For each function below: identify the key features and sketch a graph of the polynomial.

|                                                                                             |  |
|---------------------------------------------------------------------------------------------|--|
| <p>9. <math>y = (x - 1)(x + 3)^2(x + 1)</math></p>                                          |  |
| <p><i>end behavior:</i></p>                                                                 |  |
| <p><i>x-intercepts &amp; cross or touch?</i></p>                                            |  |
| <p><i>y-intercept:</i></p>                                                                  |  |
| <p>10. <math>y = x^3 - 3x^2 - 6x + 8</math></p>                                             |  |
| <p><i>end behavior:</i></p>                                                                 |  |
| <p><i>x-intercepts &amp; cross or touch?</i></p>                                            |  |
| <p><i>y-intercept:</i></p>                                                                  |  |
| <p><b>BONUS:</b><br/>Write a possible equation for the polynomial graphed to the right.</p> |  |